Superconducting diode effect

Yakov Fominov

Landau Institute for Theoretical Physics (Chernogolovka, Russia)

[1a] Ya.V. Fominov, D.S. Mikhailov, *Asymmetric higher-harmonic SQUID as a Josephson diode*, Phys. Rev. B 106, 134514 (2022).

[1b] **G.S. Seleznev**, Ya.V. Fominov, *Influence of capacitance and thermal fluctuations on the Josephson diode effect in asymmetric higher-harmonic SQUIDs*, Phys. Rev. B 110, 104508 (2024).

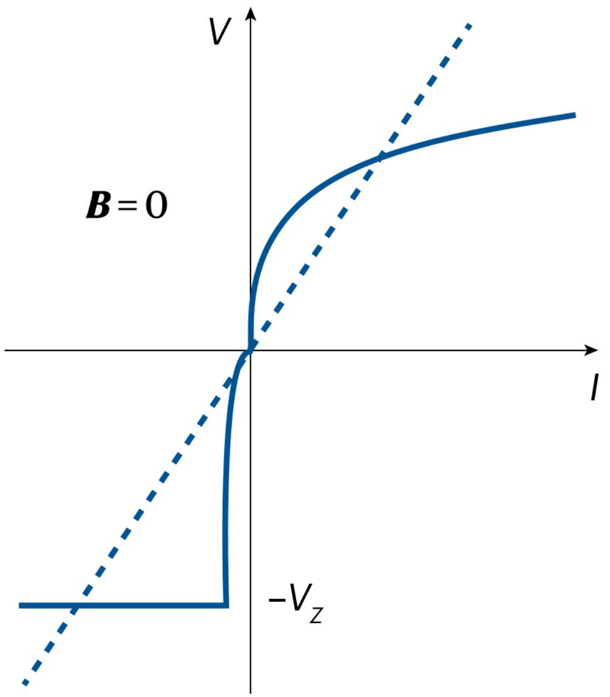
[2] D.S. Kalashnikov, **G.S. Seleznev**, A. Kudriashov, Y. Babich, D.Yu. Vodolazov, Ya.V. Fominov, V.S. Stolyarov, *Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge*, accepted to Phys. Rev. B (2025).

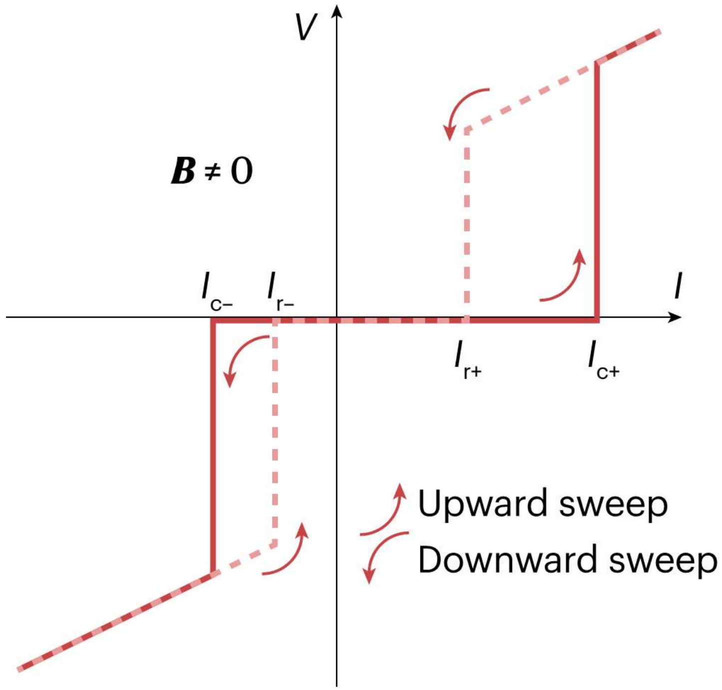
[3] **Yu.A. Dmitrievtsev**, Ya.V. Fominov, *Superconducting orbital diode effect in SN bilayers*, in preparation.

Diode

[from review Nadeem *et al.*, Nat. Rev. Phys. (2023)]

Normal vs Superconducting



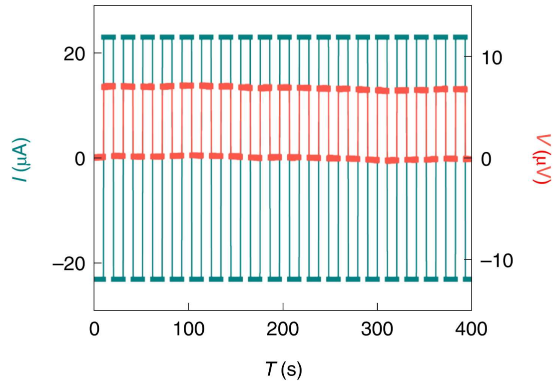


Standard ingredients of the superconducting diode effect:

- broken inversion symmetry (P-symmetry)
- broken time-reversal symmetry (T-symmetry)

Superconducting diode effect: Why so popular?

- Interesting fundamental effect depending on symmetries and, hence, probing them (e.g, diode effect at zero magnetic field $\Rightarrow$ intrinsic TRSB)

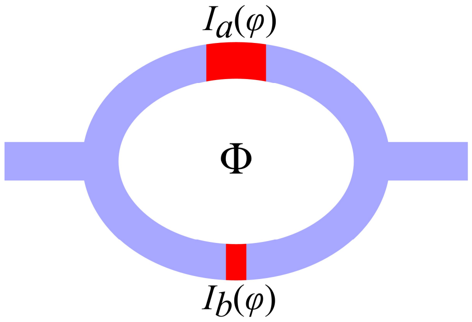


Pal *et al*., Nature Physics (2022)

- Possible applications:
  - voltage rectifiers
  - filters
  - current converters

- Diverse physical platforms and physical mechanisms:
  - ❑ bulk systems and Josephson junctions
  - vortices
  - stray fields of magnets
  - exchange field and spin-orbit coupling
  - topological materials
  - spatial symmetry breaking either on atomic level (NCS crystals) or macroscopically by heterostructure design
  - self-field effects in long Josephson junctions and in SQUIDs
  - SQUIDs with higher Josephson harmonics

1a. Josephson diode effect in asymmetric SQUIDs with higher harmonics



$$\varphi_a - \varphi_b = \phi, \qquad \phi = 2\pi \frac{\Phi}{\Phi_0}$$

CRP of the SQUID: $I_s(\varphi, \phi) = I_a(\varphi_a) + I_b(\varphi_b) = I_a(\varphi + \phi/2) + I_b(\varphi - \phi/2)$

Sinusoidal case: $I_s(\varphi, \phi) = I_{a1} \sin(\varphi + \phi/2) + I_{b1} \sin(\varphi - \phi/2) = \boxed{I_1(\phi) \sin(\varphi + \gamma)}$

where $I_1(\phi) = \sqrt{I_{a1}^2 + I_{b1}^2 + 2I_{a1}I_{b1}\cos\phi}$

$$\tan\gamma = \frac{I_{a1} - I_{b1}}{I_{a1} + I_{b1}} \tan\frac{\phi}{2}$$

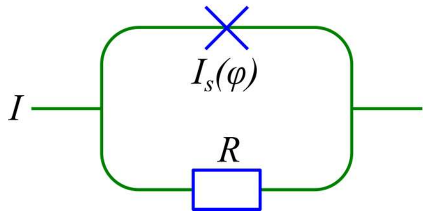
Critical currents in two directions: $I_c^+ = I_c^-$

More generally, this is so

(a) in symmetric SQUIDs with arbitrary $I_a(\varphi) = I_b(\varphi)$

(b) when $I_a(\varphi)$ and $I_b(\varphi)$ contain the same single harmonic (with arbitrary amplitudes)

Resistively-shunted junction (RSJ) model



$$\frac{V}{R} + I_s(\varphi) = I, \qquad V = \frac{\hbar}{2e}\frac{\partial \varphi}{\partial t}$$

$I_s(\varphi) = I_1 \sin\varphi$ — sinusoidal current-phase relation (CPR)

Units: $\omega_0 = 2eI_1R/\hbar, \quad \tau = \omega_0 t, \quad j = I/I_1, \quad v = V/I_1R$

Resistively-shunted junction (RSJ) model:

$$v = \dot\varphi$$

$$\dot\varphi + \sin\varphi = j$$

[Aslamazov, Larkin, Ovchinnikov (1968),
Stewart (1968),
McCumber (1968)]

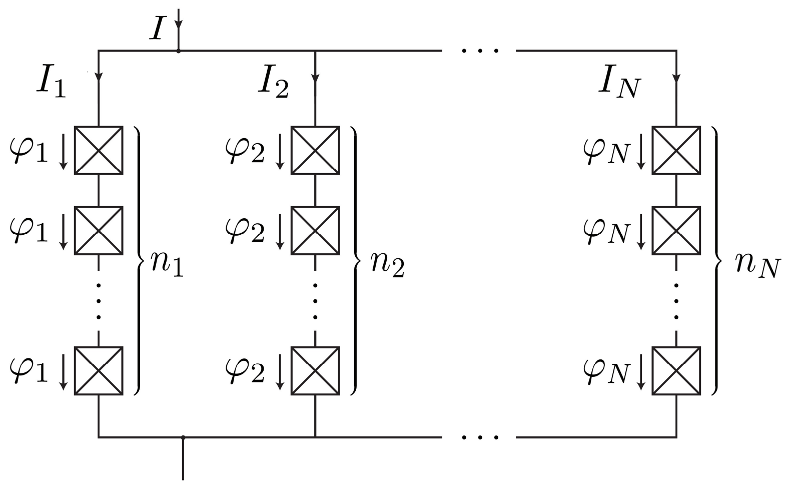
Higher Josephson harmonics

$$I_s(\varphi) = \sum_{n=1}^{\infty} I_n \sin n\varphi$$

— nonsinusoidal CPR with higher harmonics

- higher orders wrt barrier transparency
- point contacts
- SNS junctions
- *SFS junctions*
- CPR engineering
- etc.

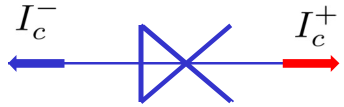
[Haenel, Can, arXiv(2022)]



RSJ equation: $\dot{\varphi} + J(\varphi) = j, \qquad J(\varphi) = I_s(\varphi)/I_1$

Asymmetric higher-harmonic SQUID as a Josephson diode

General case: $I_c^+ \neq I_c^-$



"Minimal model":

$$I_a(\varphi) = I_{a1} \sin\varphi$$

$$I_b(\varphi) = I_{b1} \sin\varphi + I_{b2} \sin 2\varphi$$

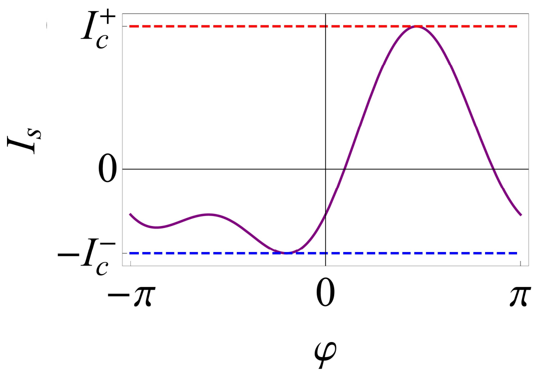
Effective Josephson junction with

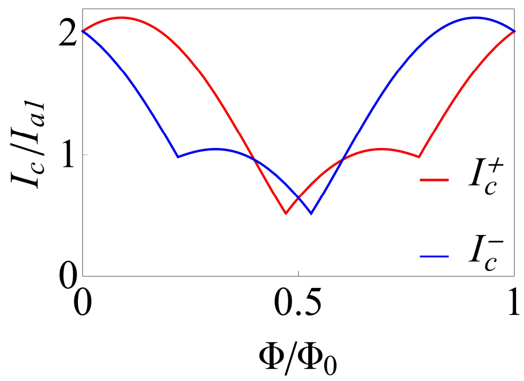
$$J(\varphi) = \sin\varphi + A \sin(2\varphi - \tilde{\phi})$$

where

$$A(\phi) = I_{b2}/I_1(\phi), \quad \tilde{\phi} = \phi + 2\gamma(\phi)$$

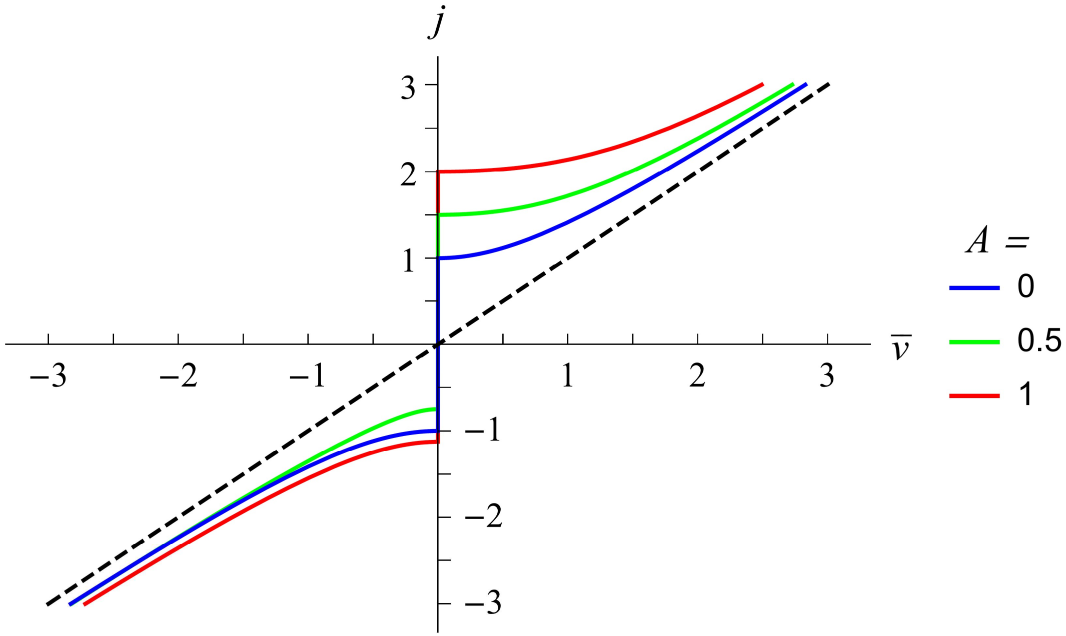
Analytics in limiting cases: diode effect at $A \sin\tilde{\phi} \neq 0$





— asymmetry of critical currents

Asymmetry of the CVC



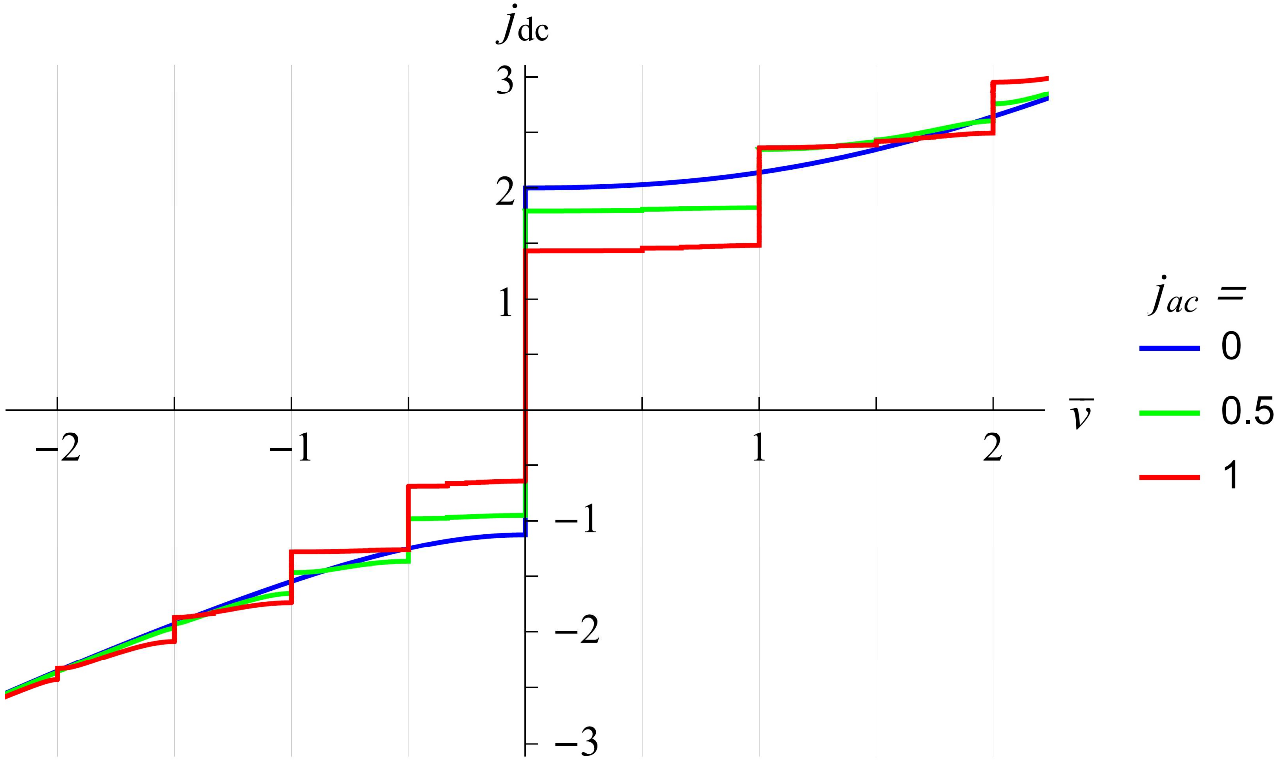
Diode effect:

$$j_c^+ \neq j_c^-$$

$$j(-\overline{v}) \neq -j(\overline{v})$$

Asymmetry of the Shapiro steps

$$j = j_{\mathrm{dc}} + j_{\mathrm{ac}} \cos \omega t$$



Diode effect: $j_c^+ \neq j_c^-$

$$j_{\mathrm{dc}}(-\overline{v}) \neq -j_{\mathrm{dc}}(\overline{v})$$

Conclusions-1a

Josephson diode effect in asymmetric SQUIDs with higher Josephson harmonics.

Minimal model: $J(\varphi) = \sin\varphi + A\sin(2\varphi - \tilde{\phi})$, at $A\sin\tilde{\phi} \neq 0$

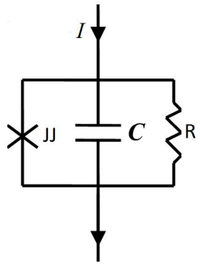
- Asymmetric critical current: $I_c^+ \neq I_c^-$
- Asymmetric current-voltage characteristic: $I(-V) \neq -I(V)$
- Asymmetric integer and fractional Shapiro steps (current-driven regime),

  $\overline{V} = \left(\frac{n}{k}\right)\frac{\hbar\omega}{2e}$
- Efficiency and polarity of the diode effect depend on the external magnetic flux

[1a] Ya.V. Fominov, D.S. Mikhailov, *Asymmetric higher-harmonic SQUID as a Josephson diode*, Phys. Rev. B **106**, 134514 (2022).

1b. Effects of capacitance and temperature

RSJ model $\Longmapsto$ RCSJ model (added capacitance and thermal fluctuations):

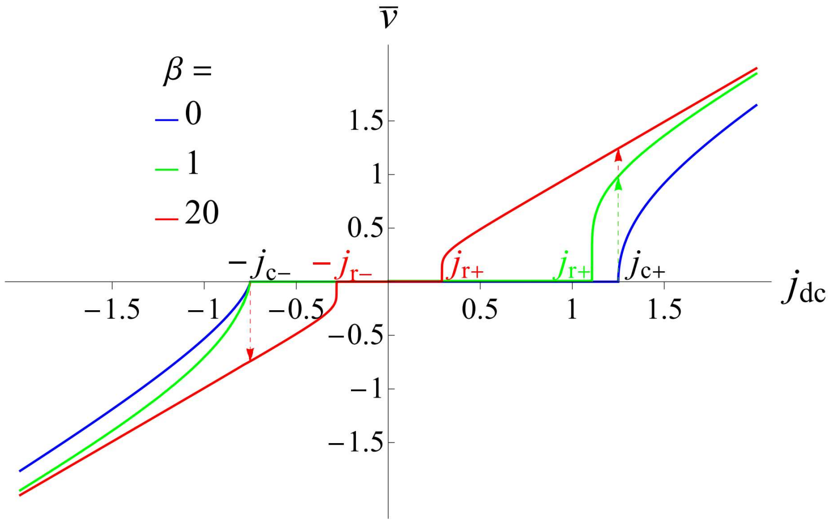


$$\frac{d^2\varphi}{d\tilde{\tau}^2} + \varepsilon \frac{d\varphi}{d\tilde{\tau}} + J(\varphi) = j_{\mathrm{dc}} + j_{\mathrm{ac}} \cos(\tilde{\omega}\tilde{\tau} + \delta) + \xi(\tilde{\tau})$$

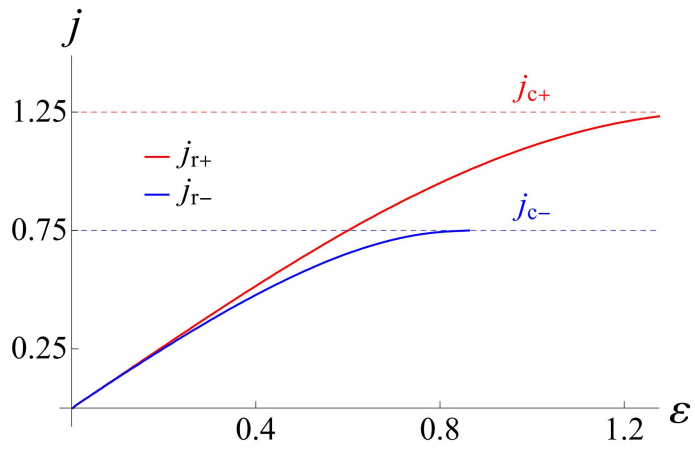
$$\varepsilon = 1/\sqrt{\beta}, \quad \beta = (2e/\hbar) I_1 R^2 C, \quad \tilde{\tau} = \omega_p t, \quad \omega_p = \sqrt{2eI_1/\hbar C}$$

McCumber parameter

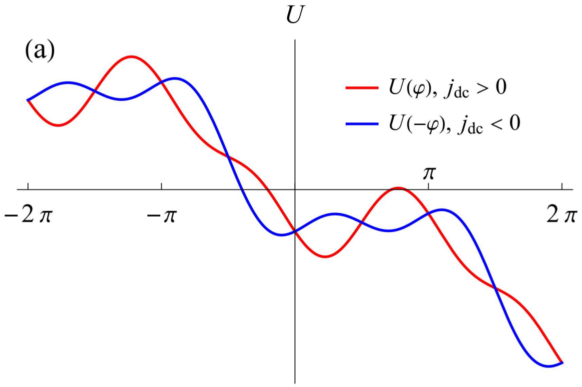
Effects of capacitance

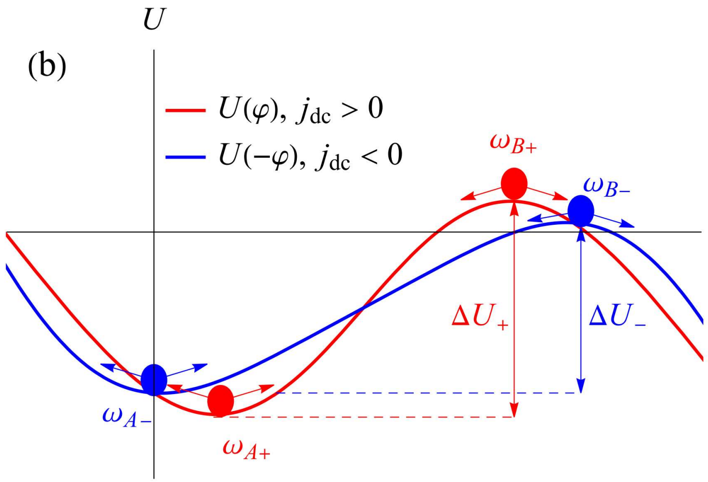


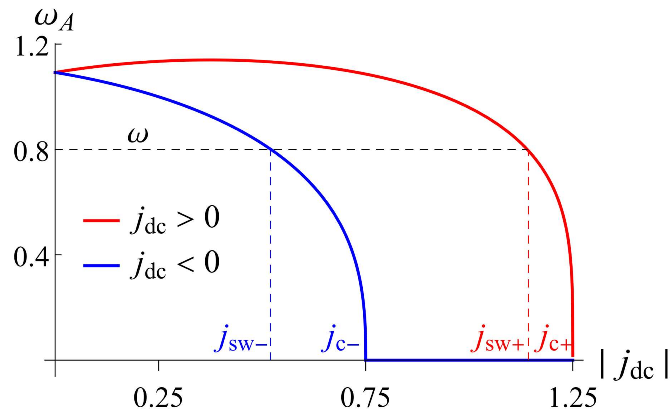
- Asymmetry in the R state is generally suppressed by capacitance (both without and with external ac irradiation)
- New effect: asymmetry of the retrapping currents $j_{r\pm}$
- Single-sides hysteresis



Asymmetry of the switching currents (S to R state)







- Effect of external ac irradiation on the S state:
  Josephson plasma resonances at $\omega = \omega_{A\pm}(j_{sw\pm})$

$$j_{\mathrm{sw}\pm} = \sqrt{1-\tilde{\omega}^4} + \frac{2A\tilde{\omega}^6 \cos\tilde{\phi}}{\sqrt{1-\tilde{\omega}^4}} \pm A(1+2\tilde{\omega}^4)\sin\tilde{\phi}$$

Effects of thermal fluctuations

$$\beta\ddot{\varphi} + \dot{\varphi} + \sin\varphi + A\sin(2\varphi - \tilde{\phi}) = j_{\rm dc} + \xi(\tau)$$

$$\langle \xi(\tau)\xi(\tau')\rangle = 2\theta\delta(\tau - \tau'), \;\text{ where } \theta = 2eT/\hbar I_1 = T/E_J$$

- Fokker-Planck equation for the distribution function in the low-temperature limit $\theta \ll \Delta U_{\pm}$ (i.e., escape time ≫ sliding time)

Generalization of:
[Stratonovich — Textbook (1961)]
[Ivanchenko, Zil'berman (1968)]
[Ambegaokar, Halperin (1969]

Zero capacitance, asymmetric CVC:

$$\langle \overline{v}_{\pm}(j_{\rm dc})\rangle = \pm 2\omega_{A\pm}\omega_{B\pm}\sinh\left(\frac{j_{\rm dc}\pi}{\theta}\right)\exp\left(\frac{-\Delta U_{\pm} - j_{\rm dc}\pi}{\theta}\right)$$

Nonzero capacitance, asymmetric escape rates:

$$\tilde{\tau}_{l\pm}^{-1} = \frac{\tilde{\omega}_{\rm att\pm}}{2\pi}\exp\left(-\frac{\Delta U_{\pm}}{\theta}\right)$$

Effects of thermal fluctuations: switching currents

Slowly increasing current: $j_{\mathrm{dc}}(\tilde{\tau}) = a\tilde{\tau}, \quad a\tilde{\tau}_{l\pm} \ll 1$

Switching: probability to remain in the potential well = 1/2

$$j_{\mathrm{sw}\pm} = j_{c\pm} - \begin{cases} \left(\frac{\theta}{u_{c\pm}} \ln \frac{2\theta\omega_{c\pm}^2}{(6\pi \ln 2)\varepsilon a u_{c\pm}} \right)^{2/3}, & \varepsilon \gg 1 \\ \left(\frac{\theta}{u_{c\pm}} \ln \frac{2\theta\omega_{c\pm}}{(6\pi \ln 2) a u_{c\pm}} \right)^{2/3}, & \varepsilon \ll 1 \end{cases}$$

Conclusions-1b

Capacitance:

- Generally, capacitance suppresses the diode effect in the R state

At the same time, qualitatively new effects:

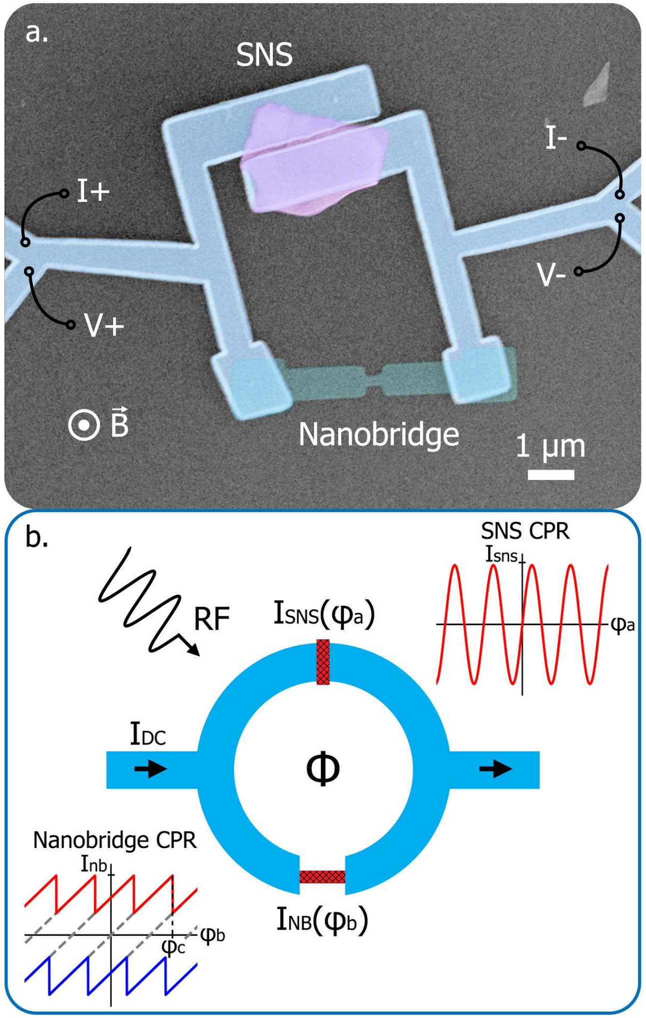
- Asymmetry of the retrapping currents
- Single-sided hysteresis
- Asymmetric switching currents due to the Josephson plasma resonances

Thermal fluctuations:

- Exponentially asymmetric CVC below the critical currents
- Asymmetry of the thermal switching currents

[1b] **G.S. Seleznev**, Ya.V. Fominov, *Influence of capacitance and thermal fluctuations on the Josephson diode effect in asymmetric higher-harmonic SQUIDs*, Phys. Rev. B **110**, 104508 (2024).

2. Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge



Center for Advanced Mesoscience & Nanotechnology

CPR of the SQUID with nanobrigde:

$$I_s(\varphi) = I_{\rm sns}(\phi)\sin(\varphi+\phi) + I_{\rm NB}(\varphi)$$

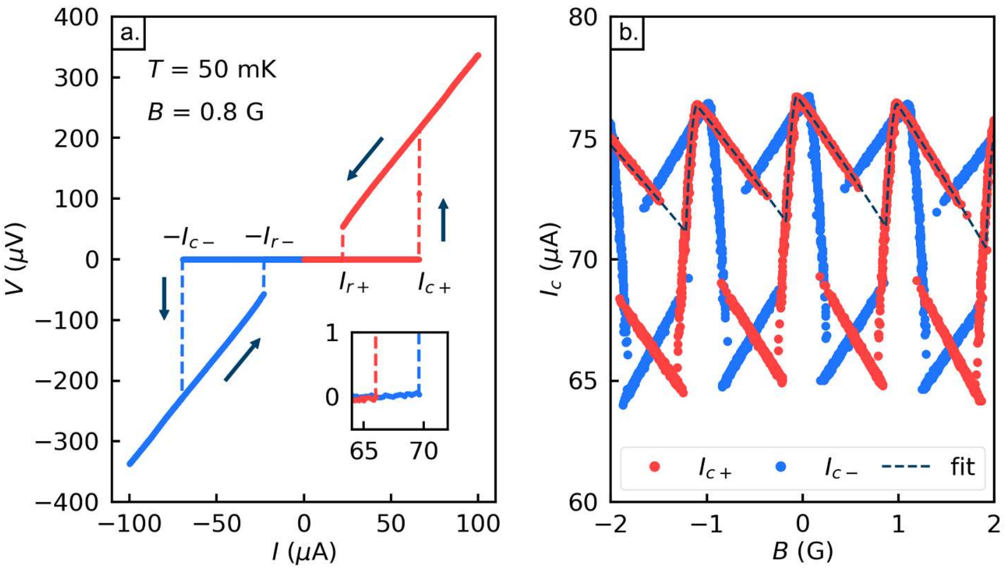
$$\frac{I_s(\varphi)}{I_{\rm nb}} = A_{0\pm} + \sum_{k=1}^{\infty} A_{k\pm}\sin\big(k(\varphi+\delta_{k\pm})\big)$$

Asymmetry of the first Josephson harmonic at $\sin\varphi_c \sin\phi \neq 0$:

$$A_{1\pm} = \sqrt{\left(\frac{I_{\rm sns}(\phi)}{I_{\rm nb}}\right)^2 + \frac{4}{\varphi_c^2} - \frac{4I_{\rm sns}(\phi)}{I_{\rm nb}\varphi_c}\cos\left(\phi \pm \varphi_c\right)}$$

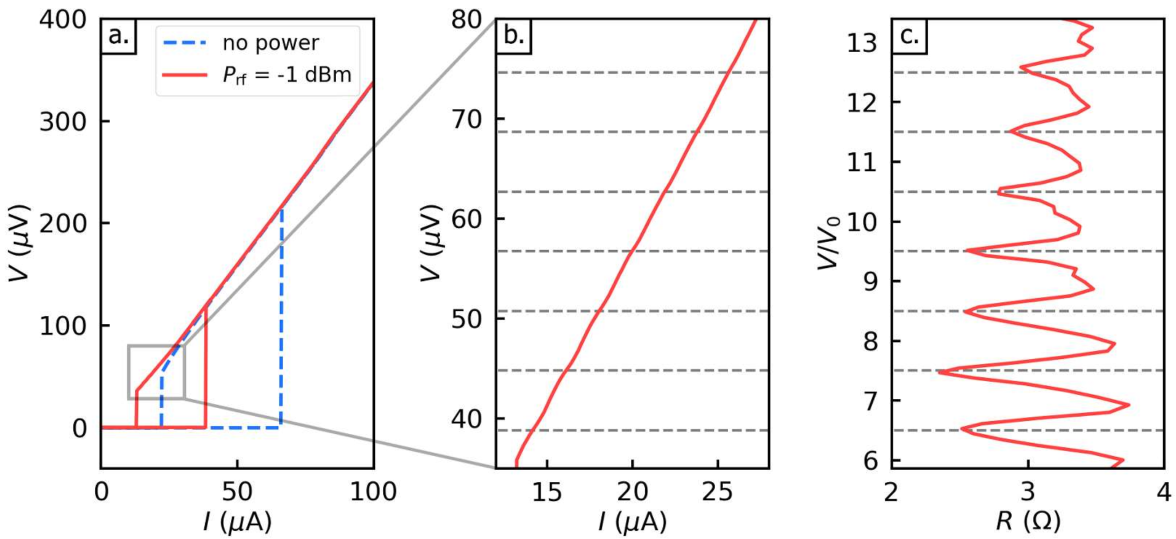
— leads to JDE!

dc and ac measurements

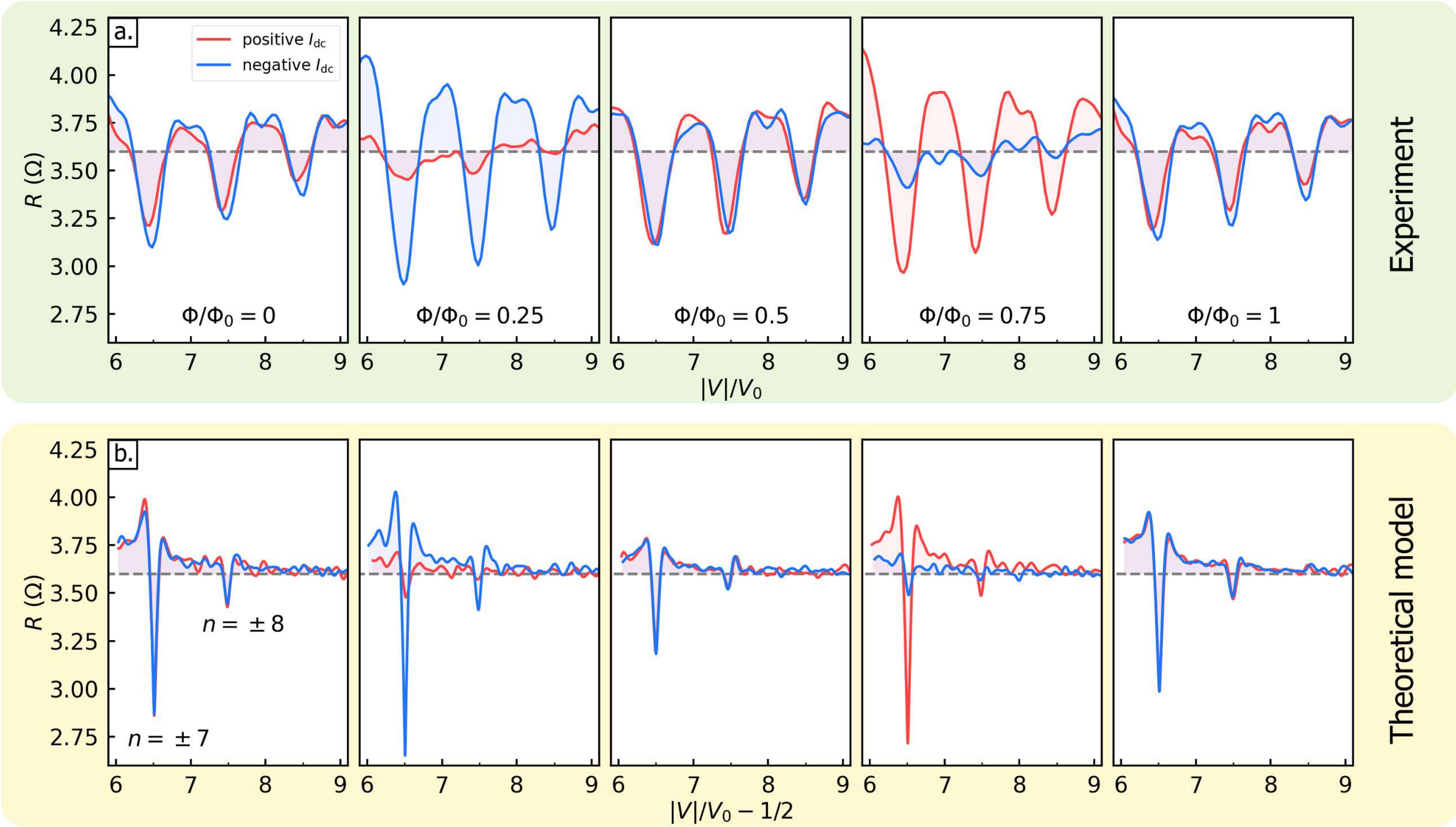


— dc measurements: asymmetry of the critical currents

— ac measurements: Shapiro steps



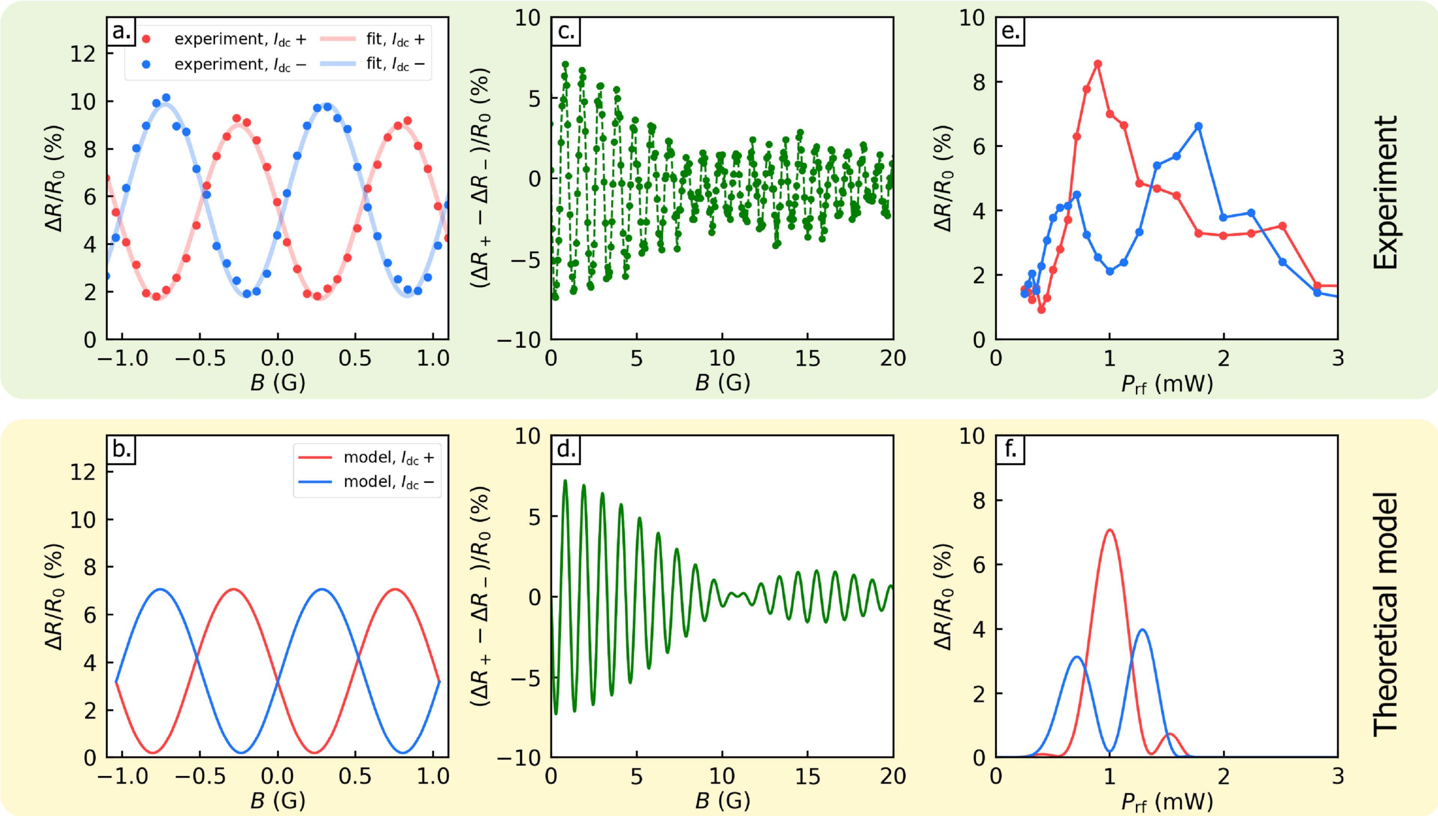
Asymmetry of Shapiro steps



Theory: RSJ model with thermal noise: $\langle \xi(\tau)\xi(\tau') \rangle = 2\mathcal{T}\delta(\tau - \tau')$

$\mathcal{T} = 2eT/\hbar I_{\mathrm{nb}}$

Analysis of the Shapiro steps



$$\mathcal{T}_\pm = \mathcal{T}/A_{1\pm} J_n(j_{ac}/\omega)$$

Theory: single-harmonic CPR.

Analytics at the centers of the Shapiro steps: $$\frac{R_\pm}{R_0} = I_0^{-2}\left(1/\mathcal{T}_\pm\right) = \begin{cases} (2\pi/\mathcal{T}_\pm)\exp(-2/\mathcal{T}_\pm), & \mathcal{T}_\pm \ll 1 \\ (1 - 1/2\mathcal{T}_\pm^2), & \mathcal{T}_\pm \gg 1 \end{cases}$$

Dependence on power of microwave irradiation: account of energy balance.

Conclusions-2

Asymmetric SQUID with nanobridge:

- Josephson diode effect manifestations in the critical currents and Shapiro steps

- New mechanism: asymmetry of the first Josephson harmonic in the effective SQUID's CPR due to (linear) multivalued CPR of the nanobridge:

  $A_{1+} \neq A_{1-}$ at $\sin\varphi_c \neq 0$

[2] D.S. Kalashnikov, **G.S. Seleznev**, A. Kudriashov, Y. Babich, D.Yu. Vodolazov, Ya.V. Fominov, V.S. Stolyarov, *Diode effect in Shapiro steps in an asymmetric SQUID with a superconducting nanobridge*, accepted to Phys. Rev. B (2025).

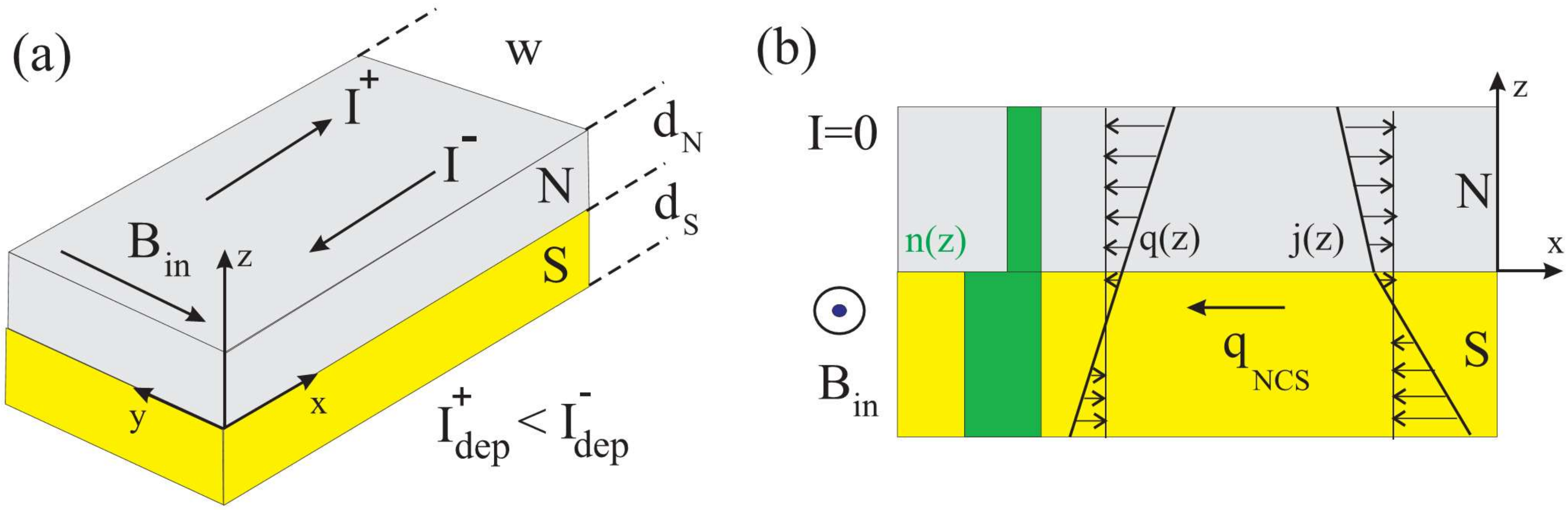
3. Superconducting orbital diode effect in SN bilayers

PHYSICAL REVIEW B **108**, 094517 (2023)

Finite momentum superconductivity in superconducting hybrids: Orbital mechanism

M. Yu. Levichev, I. Yu. Pashenkin, N. S. Gusev, and D. Yu. Vodolazov*

Institute for Physics of Microstructures, Russian Academy of Sciences, 603950 Nizhny Novgorod, GSP-105, Russia



Symmetry-breaking vector: $[\nabla n \times \mathbf{B}]$

Numerics in the case of transparent interface
+ experiment

Analytics + interface effect

Analytics: dirty-limit theory (Usadel equations) in the case of weak inhomogeneity, development of

PHYSICAL REVIEW B, VOLUME 63, 094518 (2001)

Superconductive properties of thin dirty superconductor-normal-metal bilayers

Ya. V. Fominov and M. V. Feigel'man

Weakly inhomogeneous case:

1. Thin bilayer $d \ll \xi$ + arbitrary interface transparency
2. Thick bilayer $d \geq \xi$ + opaque interface

Condition: $\frac{d^2}{\xi^2} \ll \max\left(1, \frac{R}{R_0}\right)$

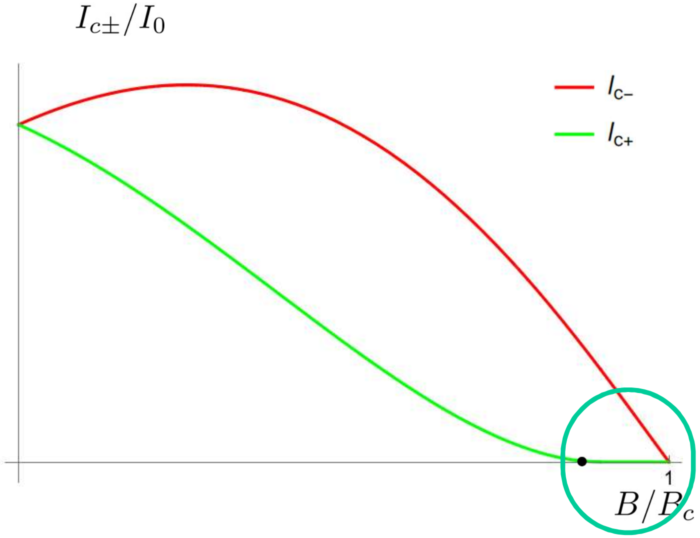
$$R_0 = R_q \frac{\delta_S + \delta_N}{|\Delta|}$$

R — interface resistance

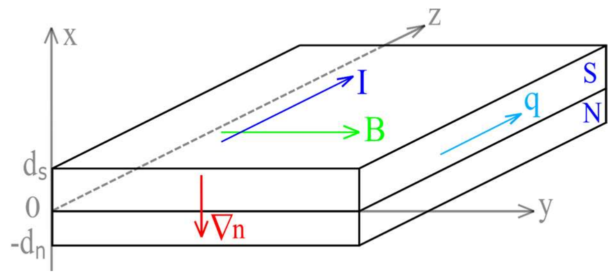
R_q — quantum resistance

$\delta_{S,N} = (\nu_{S,N} V_{S,N})^{-1}$ — level spacing

Supercurrent



$$\frac{I(q,B)}{I_0} = \frac{n_{\text{eff}}(q,B)}{n_0}\,\frac{q}{q_c} + k\left|\frac{\Delta(q,B)}{\Delta_0}\right|^2 \frac{B}{B_c}$$



— full diode effect is achievable

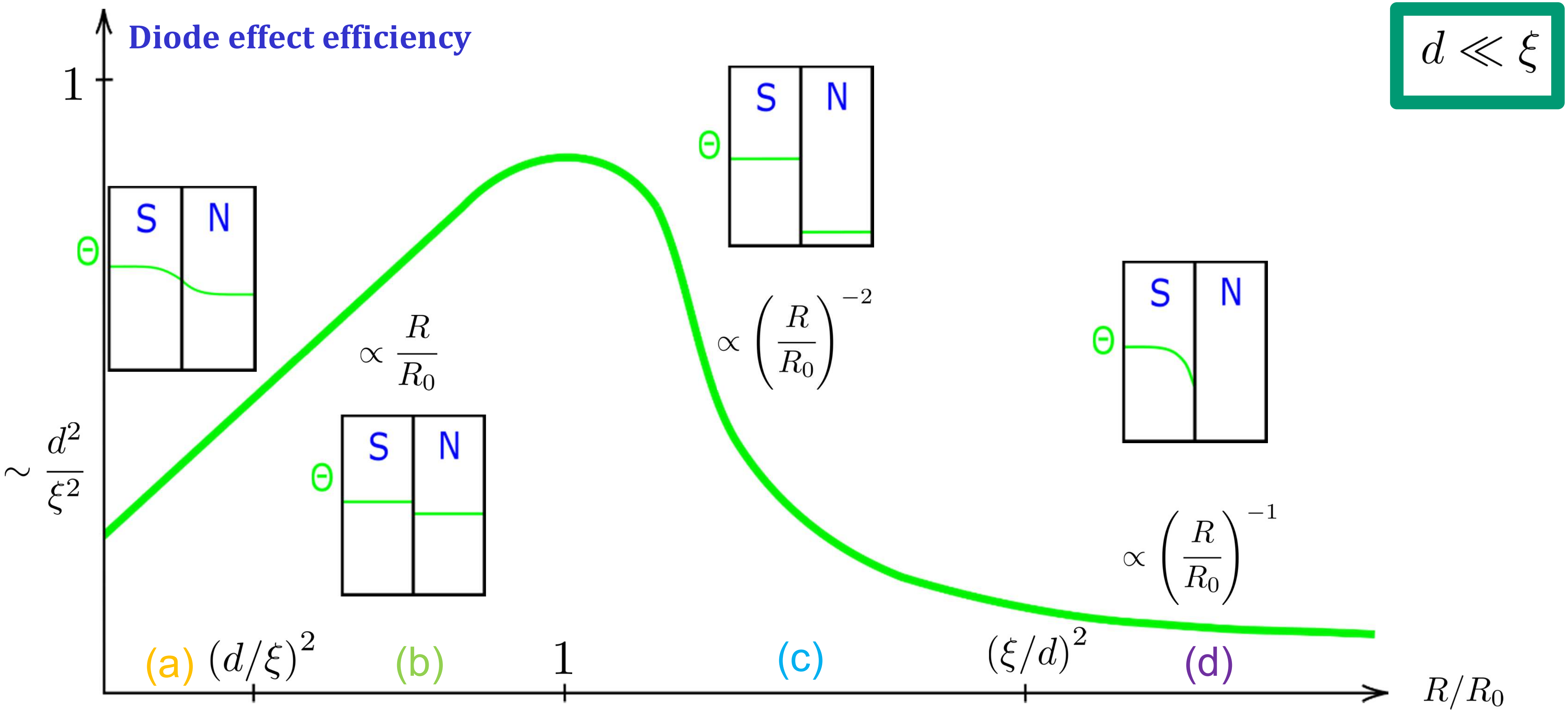
n_0, Δ_0 — superfluid density and order parameter at $T = 0$ and $B = 0$

q_c, B_c — critical momentum and magnetic field

I_0 — characteristic current value (at $n_{\text{eff}} = n_0$, $q = q_c$, $B = 0$)

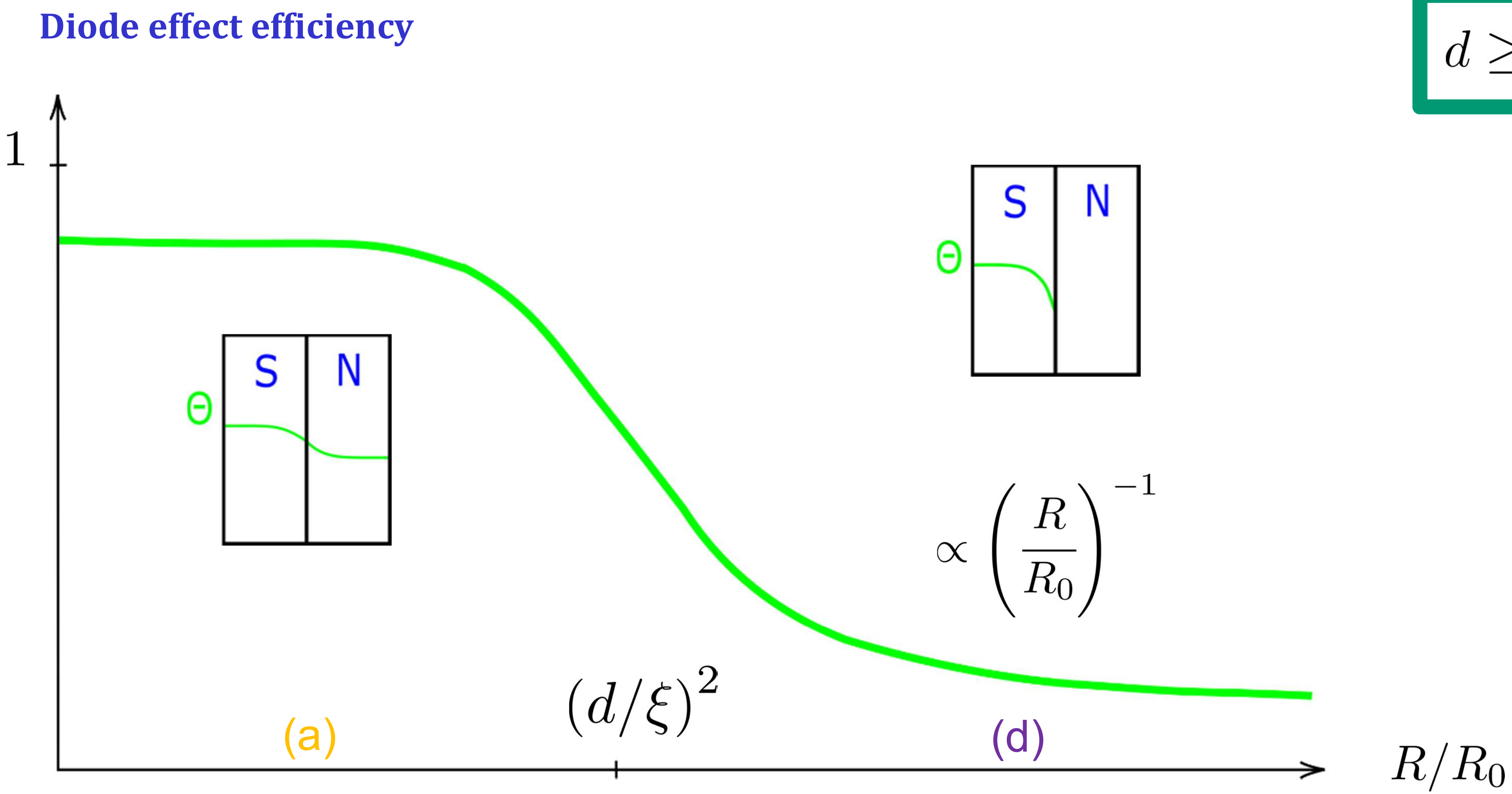
$k \ll 1$ — parameter of weak inhomogeneity

Interface effect: thin bilayer



Interface effect: thick bilayer

$d \geq \xi$



Conclusions-3

Orbital diode effect in SN bilayers:

- Analytics in weakly inhomogeneous cases $d^2/\xi^2 \ll \max(1, R/R_0)$
- Dependence of critical currents and diode efficiency on magnetic field: absolute visibility at $B \to B_c$
- Effect of transparency: nonmonotonic diode efficiency in the limit of $d \ll \xi$ with maximum at $R \sim R_0$

Poster *Tu-3* today!

Yuriy Dmitrievtsev, Ya.V. Fominov, *Superconducting orbital diode effect in SN bilayers*.

Conclusions

Superconducting diode effect:

- Fundamental effect revealing symmetries of the system

- Diverse physical platforms and physical mechanisms

❑ Work supported by the Russian Science Foundation (Grant No. 24-12-00357)